



Department of Computer Science & Mathematics

Calculus Camp

August 21-23, 2006

Exercises

Important: Do NOT use calculators!

1. Real numbers. Rational and irrational numbers. Absolute value

- 1.1. Is it possible that sum of two rational numbers is irrational? Is it possible that sum of an irrational number and a rational number is rational? Is it possible that sum of two irrational numbers is rational? Explain your answers.
- 1.2. Which of the following numbers are rational? Explain your answer.

$$1.2345, 1/2 + 1/3, \sqrt{2} + 1, 0.122222\dots, \pi + 2$$

- 1.3. Is the number $\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} - 2\sqrt{6}$ irrational? Explain your answer.
- 1.4. Let $A = \{x, y, z, 1, 2, 3\}$, $B = \{x, z, 3\}$, and $C = \{2, 4, 5\}$ Find $A \cup B$, $A \cap B$, $A \cup C$, $A \cap C$.
- 1.5. Mark the following numbers on the real line. Provide the details of your computations.

$$2, 0, \sqrt{5}, \sqrt{5} - 1, \sqrt{5} - 3, (\sqrt{5} + 1)/2$$

- 1.6. Rewrite $|\sqrt{\pi} - 2|$ without using the absolute value sign.
- 1.7. What can you say about numbers a, b, c , if $|a| + |b| + |c| = 0$? Explain your answer.
- 1.8. Solve the inequality $0 < |x - 5| < 1/2$.

2. Coordinate system on the plane. Graphs of functions

- 2.1. Mark the points with coordinates $(3, 0)$, $(1, 1)$, $(0, 3)$, $(-1, 1)$, $(-3, 0)$, $(-1, -1)$, $(0, -3)$, $(1, -1)$, $(3, 0)$, and join them by straight line segments in the given order.
- 2.2. Sketch all points on the coordinate xy -plane coordinates of which satisfy the equation $|x| + |y| = 1$.

3. Linear and quadratic functions. Polynomials

- 3.1. Find an equation of the line with y -intercept 6, parallel to the line $2x + 3y + 4 = 0$.
- 3.2. Show that the lines $3x - 5y + 19 = 0$ and $10x + 6y - 50 = 0$ are perpendicular and find their point of intersection.
- 3.3. Sketch the region bounded by the curves $y = 3x$ and $y = x^2$.
- 3.4. Solve the equation $|x^2 + x - 2| = 2$.
- 3.5. Solve the equation $|x^2 - x - 6| = x + 2$.
- 3.6. Factor $x^2 + x - 12$.
- 3.7. Factor $2x^2 - 11xy + 5y^2$.
- 3.8. Solve the equation $x^3 - 2x^2 - 5x + 6 = 0$.
- 3.9. Solve the inequality $x^3 - 2x^2 + x > 0$.
- 3.10. Find a quadratic equation with roots $1 + \sqrt{7}$ and $1 - \sqrt{7}$.

4. Fractions. Rational functions

- 4.1. Write $1.\overline{234}$ as a fraction.
- 4.2. Solve the equation $\frac{1}{x+1} = 1 - \frac{5}{2x-4}$.
- 4.3. Solve the equation $x + 3 + \frac{3}{x-1} = \frac{4-x}{x-1}$.
- 4.4. Solve the inequality $\frac{x-2}{x+2} \geq \frac{2x-3}{4x-1}$.

5. Root functions

- 5.1. Solve the equation $2\sqrt{2x^2 + 5x - 3} = 14 - 3x$.
- 5.2. Solve the equation $x - 1 = \sqrt{5 - x^2}$.
- 5.3. Solve the equation $\sqrt{10 + x} - \sqrt{19 - 3x} = 3$.
- 5.4. Solve the inequality $\sqrt{(x - 3)(2 - x)} < \sqrt{4x^2 + 12x + 11}$.
- 5.5. Solve the inequality $x + 1 > \sqrt{x + 3}$.

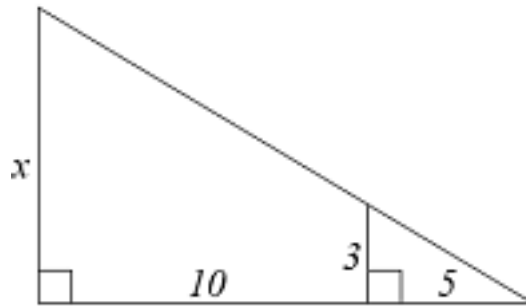
6. Trigonometric functions

- 6.1. Convert 5 radians to degrees.
- 6.2. Convert 10° to radians.
- 6.3. Prove the identity $(\sin x + \cos x)^2 = 1 + \sin 2x$.
- 6.4. Is it possible that
- (a) $\sin \alpha = .4$ and $\cos \alpha = .7$
 - (b) $\sin \alpha = \sqrt{6}/2$
 - (c) $\sin \alpha = -2/\sqrt{5}$ and $\cos \alpha = 1/\sqrt{5}$?
- 6.5. If $\sin x = \sqrt{2}/3$ and $0 < x < \pi/2$, find $\cos x$ and $\tan x$.
- 6.6. If $\cos x = -\sqrt{6}/4$ and $\pi/2 < x < \pi$, find $\sin x$ and $\tan x$.
- 6.7. If $\sin x = 1/3$ and $\sec y = 5/4$, where x and y are between 0 and $\pi/2$, evaluate $\cos(x + y)$.
- 6.8. If $\cos x = -3/2$ and $\csc y = 2$, where x and y lie between 0 and $\pi/2$, evaluate $\sin(x + y)$.
- 6.9. Find the exact value of $\cos(16\pi/3)$.
- 6.10. Find the exact value of $\sin(\pi/12)$.
- 6.11. Find the exact value of $\cos 117^\circ + \cos 63^\circ$.

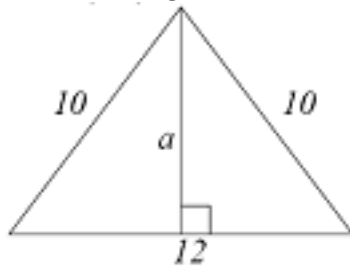
- 6.12. Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation $|\tan x| = 1$.
- 6.13. Solve the equation $2 \cos x + \sin 2x = 0$.
- 6.14. Solve the equation $2 + \cos 2x = 3 \cos x$.
- 6.15. Solve the inequality $\cos x > -1/2$.
- 6.16. Solve the inequality $\sin x \leq 1/2$.

7. Basics of plane geometry

- 7.1. Find an equation of the circle centered at $(-1, 5)$ that passes through the point $(-4, -6)$.
- 7.2. Sketch the curve given by equation $x^2 - 2x + y^2 + 2y + 1 = 0$.
- 7.3. Find the value of x in the diagram. (Note that horizontal side of the largest triangle is 15).



- 7.4. Find the length of the perpendicular bisector a in the diagram of an isosceles triangle whose sides are 10, 10, and 12.



8. General equations and inequalities

8.1. Solve for x and y :

$$\begin{cases} 2y - 4x = 10 \\ y - 2 = x^2 \end{cases}$$

8.2. Solve the inequality $3|x - 1| > (x - 1)^2 + 1$.

8.3. Solve the equation $x^4 - x^2 - 2 = 0$.

8.4. Solve the equation $2x + \sqrt{x} - 1 = 0$.

8.5. Solve the equation $\frac{4}{x + \sqrt{x^2 + x}} - \frac{1}{x - \sqrt{x^2 + x}} = \frac{3}{x}$.

8.6. Solve the equation $\sqrt{2\sqrt{7} + \sqrt{x}} - \sqrt{2\sqrt{7} - \sqrt{x}} = \sqrt[4]{28}$.

8.7. Solve the equation $\cos x = x^2 + 1$.

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